MATH 3070
Assignment # 2
Due Thursday, September 25, 2008

1. Primality in arithmetic progressions.

   (a) Prove that no consecutive terms can be both prime in any arithmetic progression with odd common difference unless one of the terms is 2.

   (b) Prove that no three consecutive terms can all be primes in any arithmetic progression with common difference of 2 except for $3, 5, 7$.

   (c) Prove that no four consecutive terms can be pairwise coprime in any arithmetic progression with odd common difference.

   (d) Exhibit 1000 pairwise coprime numbers in arithmetic progression.

2. Define the Fibonacci sequence F_n in the usual way: $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Prove that consecutive Fibonacci numbers are always relatively prime. That is, for all $n \geq 0$, $(F_n, F_{n+1}) = 1$.

3. Prove that there are infinitely many primes of the form $3n + 2$.

4. Prove that no 4th power is 2 more than a 6th power. (That is, $x^4 = y^6 + 2$ has no solutions in integers x and y.)

5. Calculate the two rightmost digits of 4^{2008}.

6. Find all values of x that satisfies each congruence.

   (a) $3x \equiv 1 \pmod{157}$

   (b) $7x \equiv 12 \pmod{36}$

Practice problems (not to be handed in):
2.1 # 25, 26, 29
2.2 # 9, 13, 14, 22
3.1 # 21
3.7 # 1, 2
5.1 # 11, 12, 13, 21