MATH 3070
Assignment # 1
Due Tuesday, September 16, 2008

Prove or disprove the following:

1. For $a,\ b \in \mathbb{Z}$, if $a|b$ and $b|a$ then $a = b$.
2. If x and y are relatively prime, then for any $n \in \mathbb{Z}$ there are integers a and b such that $ax + by = n$.
3. For any $d \in \mathbb{N}$, if $d|ab$ then either $d|a$ or $d|b$.
4. n and $2n + 1$ are always relatively prime.
5. If $(a, b) = (b, c) = 1$, then $(a, c) = 1$.
6. For all $n \in \mathbb{N}$, there is some positive integer k such that $kn + 1$ is composite.
7. For integers $a, b,$ and c, define $\gcd(a, b, c) := \max\{d \in \mathbb{N} : d|a, d|b, \text{and } d|c\}$.
   Then $\gcd(a, b, c) = \gcd(\gcd(a, b), c)$.

For each of the following pairs (x, y), use the Euclidean Algorithm to find a pair (a, b) such that $ax + by = 1$.

8. $(12, 23)$
9. $(15, 44)$

Finally...

10. Alice and Bob are consultants. Since Alice has a PhD (and Bob doesn't), Bob needs to lower his prices in order to compete. Suppose Alice charges $273 per hour and Bob charges $161 per hour. Is it possible for one of them to have made exactly $14 more than the other some time during the month? If so, how many hours does each have to work? (Assume, of course, that they bill in whole hours only)