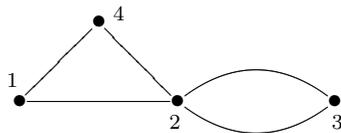


MATH 2113 / CSCI 2113
Assignment # 4
Due Monday, March 12, 2007

Each problem is worth 10 points, for a total of 70 points.

1. Section 11.1, Problems 6 and 7.
2. Section 11.1, Problems 17 and 18.
3. In a tournament with 9 players, each match pits some pair of players against each other, and no two people play each other more than once.
 - (a) Is it possible that every player plays in exactly 3 matches?
 - (b) What is the maximum possible total number of matches?
 - (c) If the tournament has a total of 19 matches, prove that some player played in at least 5 matches.
4. Prove that if a graph G is isomorphic to its complement, then $|V(G)| \equiv 0$ or $1 \pmod{4}$.
5. Prove that in a any tournament (K_n with a direction for every edge), there exists a vertex that can reach every other vertex on a path of length at most 2.
6. For which values of n does K_n contain an Eulerian circuit? How about an Eulerian trail?
7. Prove that a connected graph G has an Eulerian **trail** but no Eulerian **circuit** if and only if exactly two vertices have odd degree. (Hint: To construct the Eulerian trail, modify the graph so you can construct an Eulerian circuit, then build your trail from this circuit.)
8. (Not to be handed in) In the movie “Good Will Hunting”, there is an early scene in which Professor Lambeau (Stellan Skarsgård) says to the students of his (what appears to be) graduate level Fourier Analysis class, “. . . I also put an advanced Fourier system on the main hallway chalkboard. I’m hoping that one of you might prove it by the end of this semester. The person to do so will not only be in my good graces, but also go on to fame and fortune. . .” Here is what’s actually on that board in the movie.

Given the graph



Find:

- (a) The adjacency matrix A .
- (b) The matrix giving the number of 3 step walks.
- (c) The generating function for walks from point $i \rightarrow j$.
- (d) The generating function for walks from points [sic] $1 \rightarrow 3$.

Of course, this is a graph theory problem and has nothing to do with Fourier analysis whatsoever. There is also nothing to “prove”. The last two parts should say “generating function for *the number of* walks”. Note also that the answer Will Hunting (Matt Damon) wrote on the board is in fact correct.

It is very easy to determine the number of walks from the adjacency matrix A . Write down the answers to this problem. What can you say about the writers for the movie?