# MATH 2113 / CSCI 2113
# Assignment # 1
# Due Friday, January 19, 2007

1. In British Columbia, instead of adding TAG to a 6/49 ticket, you can play a lottery called Extra instead. In this lottery, 4 numbered balls are drawn, without replacement, from balls labelled 1 through 99. How many different Extra tickets can there be?

2. How many positive integers less than $100,000$ contain exactly one 5 and one 7?

3. Suppose you flip a fair coin $n$ times. What is the probability that the number of heads and tails are equal?

4. Count the subsets of $[n]$ that contain at least one odd number.

5. Consider a dial having a pointer that is equally likely to point to each of $n$ regions numbered 1, 2, $\ldots$, $n$. When we spin the dial three times, what is the probability that the sum of the selected numbers is $n$?

6. Find the number of solutions in non-negative integers $(x_1, x_2, \ldots, x_r)$ such that

$$x_1 + x_2 + \cdots + x_r \leq k.$$

7. A *three-of-a-kind* is a set of 5 cards with exactly three of the cards having the same rank, and the other two do *not* make a pair. For example, 3-A-3-J-3. What is the probability of being dealt a three-of-a-kind?

8. On a test, there are two sections. Section A has 10 problems and section B has 15 problems. The instructions indicate that you are to do 6 problems from section A and 10 problems from section B. In how many ways can the test be done? (Assume that all 16 required problems are done)

9. Suppose that instead of 6 problems from section A and 10 from section B as in the previous question, you are now required to do a total of 12 problems with at least 3 and at most 6 from section A. In how many ways can the test be done?

10. Prove combinatorially that

$$\sum_{k=0}^{r} \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}.$$

This formula is called the Chu-Vandermonde convolution.