MATH 2112 / CSCI 2112
Assignment # 10
Due Wednesday, November 29, 2006

Section 5.1: # 11

Section 5.2: # 17, 24

Section 5.3: # 10, 12, 17

Section 5.4 # 6

Also do the following problems. Note that problems 2 and 3 will not be marked.

1. Professor Evil's janitorial staff are each assigned several different cleaning duties. On any given day, there are 25 assigned to clean the bathrooms, 100 assigned to clean the hallways, and 50 assigned to trash collection. Usually, 35 staff are on both bathroom and hallway duty, 30 staff are on both hallway and trash duty, and 10 are assigned to both bathroom and trash. Each day, there is one unlucky henchman who gets assigned to all three chores. Use the inclusion-exclusion principle to determine the total number of janitorial staff Professor Evil has.

2. Determine whether or not the following functions are one-to-one, and whether or not they are onto. As usual, prove (or give reasons for) your answers.

   (a) $f : \mathbb{N} \to \mathbb{N}, f(x) = x^2$
   (b) $f : \mathbb{Z} \to \mathbb{N}, f(x) = x^2$
   (c) $f : \mathbb{R} \to \mathbb{Z}, f(x) = \lfloor x \rfloor$
   (d) $f : \mathbb{N} \to \mathbb{N}, f(x) = 2x$
   (e) $f : \mathbb{Q} \to \mathbb{Q}, f(x) = 2x$

3. Define a relation $\sim$ on $\mathbb{Z} \times \mathbb{Z}$ by the following rule:
   (m, n) is related to (m', n'), or $(m, n) \sim (m', n')$, if and only if either

   a. n and n' are both non-zero and the ratios m/n, m'/n' are equal, or
   b. $n = n' = 0$.

   Prove that $\sim$ is an equivalence relation on $\mathbb{Z} \times \mathbb{Z}$.