MATH 2112 / CSCI 2112
Assignment # 5
Due Wednesday, October 25, 2006

Section 10.4: # 17, 32

1. Solve for x:
   (a) $3x + 1 \equiv 10 \pmod{25}$
   (b) $7x \equiv 12 \pmod{144}$
2. Let $n = 10^k a_k + \cdots + 10a_1 + a_0$ be the base-10 digital representation of n. (That is, $0 \leq a_i \leq 9$ and $a_k \neq 0$). Prove the divisibility rule for 11. That is, n is divisible by 11 if and only if the alternating digital sum $a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^k a_k$ is divisible by 11.
3. Find the one's digit of the following:
   (a) 19^{2006}
   (b) $3^{25} \times 2^{36}$
4. (a) Prove that if x is a perfect square, then x must be congruent to 0, 1, or 4 $\pmod 8$.
   (b) Prove that numbers of the form $8k + 7, k \in \mathbb{Z}$ cannot be written as a sum of 3 squares.
5. Prove that $\sqrt{7}$ is irrational.
6. If a and b are both irrational, must $a + b$ also be irrational?
7. If a and b are both irrational and both positive, must $a + b$ also be irrational?
8. If a and b are both irrational, must ab also be irrational?
9. If a is irrational and positive, must $\sqrt{a}$ also be irrational?

Bonus Problem: (due Wednesday, Nov 1)

Prove that a number is rational if and only if its decimal expansion either terminates or eventually repeats by proving the following.

(a) Prove that if the decimal expansion of a number terminates, then it is rational.

(b) Prove that if the decimal expansion of a number eventually repeats, then it is rational.

(c) Prove that if the denominator of a rational number is of the form $2^a 5^b$, then its decimal expansion terminates.

(d) Prove that if the denominator of a rational number is relatively prime to 10, then it can be expressed in the form

$$\frac{x}{99\ldots99}$$

for some string of 9's in the denominator, and show that the decimal expansion of this fraction repeats.

(e) Prove that every rational number has a decimal expansion that either terminates or eventually repeats.