MATH 2112 / CSCI 2112
Assignment # 4
Due Wednesday, October 11, 2006

Prove or disprove the following:

1. For all $n \in \mathbb{N}$, there is some positive integer k such that $kn + 1$ is composite.
2. If x and y are relatively prime, then for any $n \in \mathbb{Z}$ there are integers a and b such that $ax + by = n$.
3. For any $d \in \mathbb{N}$, if $d|ab$ then either $d|a$ or $d|b$.
4. If a prime p divides a perfect square, then so does p^2.
5. n and $n + 1$ are always relatively prime.
6. n and $2n + 1$ are always relatively prime.
7. For all $n \in \mathbb{N}$, if $3 \nmid n$, then n can be expressed in the form $3k + 1$ or $3k + 2$ for some integer k.
8. If $(a, b) = (b, c) = 1$, then $(a, c) = 1$.

Use the Euclidean Algorithm to find the gcd of the following pairs of integers. Show all work.

9. $(142, 34)$
10. $(31415926, 2718281)$
11. $(654321, 123456)$

For each of the following pairs (x, y), use the Euclidean Algorithm to find a pair (a, b) such that $ax + by = 1$.

12. $(12, 23)$
13. $(15, 44)$

Finally...

14. Alice and Bob are consultants. Since Alice has a PhD (and Bob doesn't), Bob needs to lower his prices in order to compete. Suppose Alice charges $273 per hour and Bob charges $161 per hour. Is it possible for one of them to have made exactly $14 more than the other some time during the month? If so, how many hours does each have to work? (Assume, of course, that they bill in whole hours only)